Lines and Planes

Old Stuff:

Given vectors $\vec{u} = (x_1, y_1, z_1)$ and $\vec{v} = (x_2, y_2, z_2)$ and points $P(a_1,b_1,c_1)$ and $Q(a_2,b_2,c_2)$

$\vec{u} \cdot \vec{v} = (x_1 x_2 + y_1 y_2 + z_1 z_2)$

$= |\vec{u}|\,|\vec{v}| \cos\theta$

$\vec{u} \times \vec{v} = ((y_1 z_2 - y_2 z_1), -(x_1 z_2 - x_2 z_1), (x_1 y_2 - x_2 y_1))$ Use the MAGIC method here!!!

$\vec{u} \times \vec{v}$ will **always** be perpendicular to both $\vec{u}$ and $\vec{v}$

$\overrightarrow{PQ} = (a_2 - a_1 , b_2 - b_1 , c_2 - c_1)$

Vectors are parallel iff they are scalar multiples of each other
Vectors are perpendicular iff their dot product is zero.

Lines

Lines in 2-space: given a point $P(x_0 , y_0)$ and a direction vector $\vec{d} = (d_1 , d_2)$

Vector equation: $\vec{r} = (x_0 ,y_0) + t(d_1 ,d_2)$

Parametric equations: $x = x_0 + td_1$

$y = y_0 + td_2$

Symmetric equation: $\dfrac{x - x_0}{d_1} = \dfrac{y - y_0}{d_2}$

Scalar equation: $Ax + By + C = 0$ where is $\vec{n} = (A , B)$ normal (perpendicular) to the line

Lines in 3-space: given a point $P(x_0 , y_0 , z_0)$ and a direction vector $\vec{d} = (d_1 , d_2 , d_3)$

Vector equation: $\vec{r} = (x_0 ,y_0 ,z_0) + t(d_1 ,d_2, d_3)$

Parametric equations:

$x = x_0 + td_1$

$y = y_0 + td_2$

$z = z_0 + td_3$

Symmetric equation: $\frac{x - x_0}{d_1} = \frac{y - y_0}{d_2} = \frac{z - z_0}{d_3}$

Scalar equation: NOT POSSIBLE

To find the point of intersection of two lines:

1) work with the scalar equations (as in grade 10) or
2) work with parametric equations, making sure to check your solution in the z co-ordinate!

The possibilities given two lines are:

Parallel and distinct
Intersect at exactly one point
Coincident
Skew

Planes

Given a point $P(x_0, y_0, z_0)$ and a direction vectors $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$

Vector equation: $\vec{r} = (x_0, y_0, z_0) + t(a_1, a_2, a_3) + s(b_1, b_2, b_3)$

$$x = x_0 + ta_1 + sb_1$$

Parametric equations: $y = y_0 + ta_2 + sb_2$

$$z = z_0 + ta_3 + sb_3$$

Scalar equation: $Ax + By + Cz + D = 0$ where $\vec{n} = (A, B, C)$ is normal to the plane.

Intersections

Lines and Planes: Plug the parametric equation of the line into the scalar equation of the plane.

2 Planes: Work from the scalar equations using elimination or substation, the answer will either be

(1) the are the same plane (coincident)
(2) the have a line of intersection
(3) they have no intersection (hence parallel and distinct)

3 Planes: Again work from the scalar equations using elimination/substation or row reduction (in a matrix). You can check the scalar triple product first.