All about Trig!

Angle Measurement:

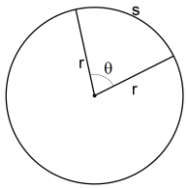
π radians = 180°

To convert from rads to degrees multiply by: $\frac{180^\circ}{\pi}$

To convert from degrees to rads multiply by: $\frac{\pi}{180}$

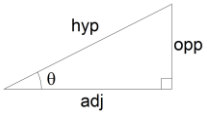
$1^\circ = \frac{\pi}{180}$ rad 1 rad = $\frac{180^\circ}{\pi}$

$\theta = \frac{s}{r}$ (θ in radians)



Right Angle Trigonometry

SOHCAHTOA



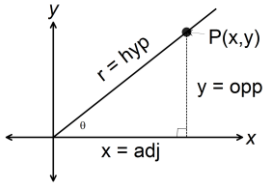
$\sin\theta = \frac{opp}{hyp}$ $\csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\sec\theta = \frac{hyp}{adj}$

$\tan\theta = \frac{opp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Trigonometric Functions

$r = \sqrt{x^2 + y^2}$



$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

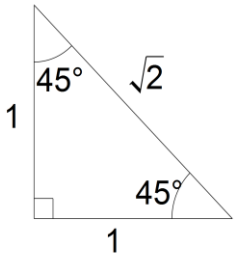
$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{x}{r}$

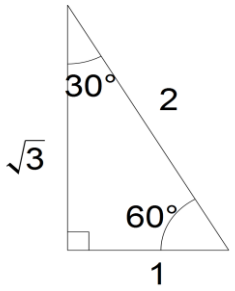
$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

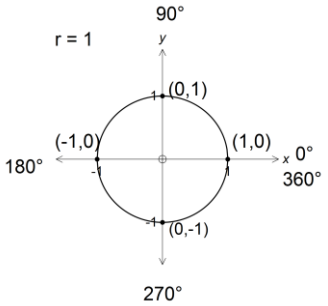
Trigonometric Values of Important Angles

DO NOT memorize this chart; use the special triangles, related acute angles, and the unit circle instead.

θ degrees)	θ (radians)	sinθ	cosθ	tanθ
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	DNE

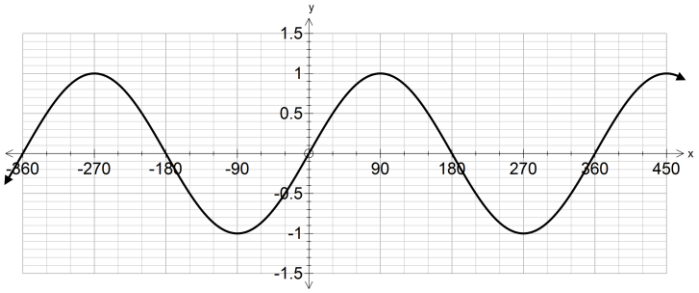






Graphing Sinusoidal Functions

$y = \sin x$

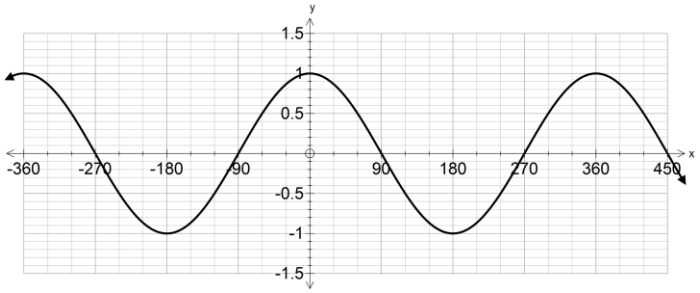


x	$\sin x$
0°	0
90°	1
180°	0
270°	-1
360°	0

Amplitude: 1
Period: 360°
Domain: $\{x \in R\}$
Range: $\{y \in R \mid -1 \leq y \leq 1\}$

x - intercepts: (0, 0), (180, 0), (360,0)
x –intercepts occur at any multiples of 180
y – intercept: (0,0)
Max Value: 1
Max Point: (90, 1)
Max points occur at (90 + any multiple of 360)
Min Value: -1
Min Point: (270, -1)
Min points occur at (270 + any multiple of 360)

$y = \cos x$



x	$\cos x$
0°	1
90°	0
180°	-1
270°	0
360°	1

Amplitude: 1
Period: 360°
Domain: $\{x \in R\}$
Range: $\{y \in R \mid -1 \leq y \leq 1\}$

x - intercepts: (90, 0), (270, 0)
x –intercepts occur at (90 + any multiple of 180)
y – intercept: (0,1)
Max Value: 1
Max Point: (0, 1), (360, 1)
Max points occur at any multiple of 360
Min Value: -1
Min Point: (180, -1)
Min points occur at (180 + any multiple of 360)

Transformations

$y = a\sin(k(x - d)) + c$ or $y = a\cos(k(x - d)) + c$

$|a|$ = amplitude $= \frac{\max - \min}{2}$, reflection in x – axis if $a < 0$

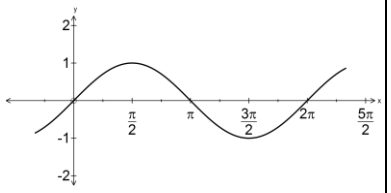
$\frac{360}{|k|}$ = period , $\frac{360}{\text{period}} = k$, reflection in y – axis if $k < 0$

d = phase shift (= distance to first max point for a cosine graph)

c = vertical translation $= \frac{\max + \min}{2}$
also known as the equilibrium line, or the axis of the curve

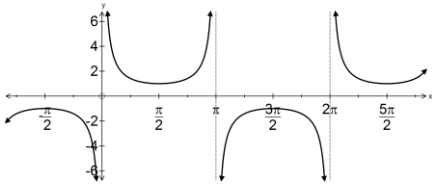
Graphs of the Trigonometric Functions

$y = \sin x$



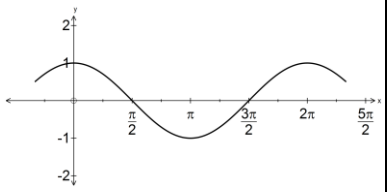
Amplitude: 1
Period: 2π
Domain: $\{x \in R\}$
Range: $\{y \in R \mid -1 \le y \le 1\}$
x - intercepts: $\{\pi n \mid n \in I\}$
y – intercept: (0,0)
Max Value: 1 when: $x \in \left\{ \frac{\pi(4n+1)}{2} \mid n \in I \right\}$
Min Value: -1 when: $x \in \left\{ \frac{\pi(4n-1)}{2} \mid n \in I \right\}$
Asymptotes: N/A

$y = \csc x$



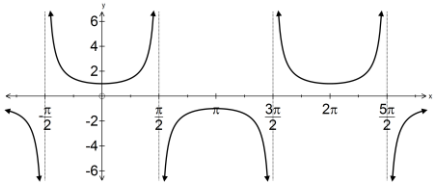
Amplitude: N/A
Period: 2π
Domain: $\{x \in R \mid x \neq n\pi,\ n \in I\}$
Range: $\{y \in R \mid |y| \ge 1\}$
Intercepts: N/A
Max Value: - 1, when: $x \in \left\{ \frac{\pi(4n-1)}{2} \mid n \in I \right\}$
Min Value: 1, when: $x \in \left\{ \frac{\pi(4n+1)}{2} \mid n \in I \right\}$
Asymptotes: $x = n\pi,\ n \in I$

$y = \cos x$



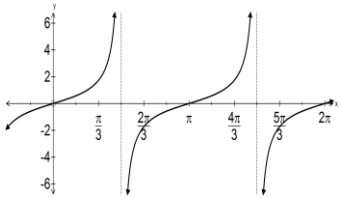
Amplitude: 1
Period: 2π
Domain: $\{x \in R\}$
Range: $\{y \in R \mid -1 \le y \le 1\}$
x - intercepts: $\left\{ \frac{(2n+1)\pi}{2} \mid n \in I \right\}$
y – intercept: (0,1)
Max Value: 1 when: $x \in \{2n\pi \mid n \in I\}$
Min Value: -1 when: $x \in \{(2n+1)\pi \mid n \in I\}$
Asymptotes: N/A

$y = \sec x$



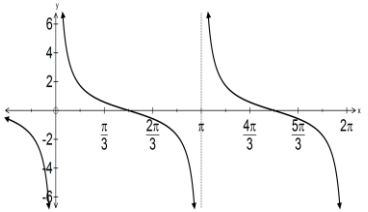
Amplitude: N/A
Period: 2π
Domain: $\left\{ x \in R \;\middle|\; x \neq \frac{(2n+1)\pi}{2},\ n \in I \right\}$
Range: $\{y \in R \mid |y| \ge 1\}$
Intercepts: (0,1)
Max Value: -1, when: $x \in \{(2n+1)\pi \mid n \in I\}$
Min Value: 1, when: $x \in \{2n\pi \mid n \in I\}$
Asymptotes: $x = \frac{(2n+1)\pi}{2},\ n \in I$

$y = \tan x$



Amplitude: N/A
Period: π
Domain: $\left\{ x \in R \;\middle|\; x \neq \frac{(2n+1)\pi}{2}, n \in I \right\}$
Range: $\{y \in R\}$
x – intercepts: $\{\pi n \mid n \in I\}$
y – intercept: (0, 0)
Max Points: N/A
Min Points: N/A
Asymptotes: $x = \frac{(2n+1)\pi}{2},\ n \in I$

$y = \cot x$



Amplitude: N/A
Period: π
Domain: $\{x \in R \mid x \neq n\pi,\ n \in I\}$
Range: $\{y \in R\}$
x – intercepts: $\left\{ \frac{(2n+1)\pi}{2} \mid n \in I \right\}$
y – intercept: (0, 0)
Max Points: N/A
Min Points: N/A
Asymptotes: $x = n\pi,\ n \in I$

Fundamental Identities

You MUST KNOW THESE:

Quotient Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}\ ,\ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta}\ ,\ \sec\theta = \frac{1}{\cos\theta}\ ,\ \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identity:

$$\sin^2\theta + \cos^2\theta = 1 \ \therefore\ 1 - \sin^2\theta = \cos^2\theta \text{ and } 1 - \cos^2\theta = \sin^2\theta$$

These can be useful:

$$\cos(-\theta) = \cos\theta,\quad \sin(-\theta) = -\sin\theta,\quad \tan(-\theta) = -\tan\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta,\ \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta,\ \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$1 + \tan^2\theta = \sec^2\theta,\quad 1 + \cot^2\theta = \csc^2\theta$$

Addition/Subtraction

$$\sin(x + y) = \sin x\cos y + \cos x\sin y$$
$$\sin(x - y) = \sin x\cos y - \cos x\sin y$$

$$\cos(x - y) = \cos x\cos y + \sin x\sin y$$
$$\cos(x + y) = \cos x\cos y - \sin x\sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

Double Angles

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half Angles

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Formulas

Law of Sines

Use for AAS, ASA or SSA
(watch for the ambiguous case with SSA)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

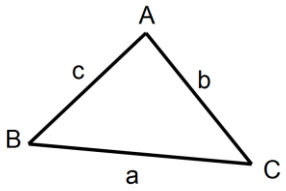
Law of Cosines

Use for SAS:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



Use for SSS:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$